Differential Equations

HW #0 due tonight at midnight

HW #1 due Friday at midnight

Office Hours: Tues 1:30pm - 2:30 pm Est

Thurs 3:00pm - 5:00pm

Same Zoom link

2.1 First order

Initial Value Problem (IVP)

$$\frac{dy}{dt} = y - t$$

ordinary differential equation

$$\left.\begin{aligned} y' &= y^2 - t \\ y' &= e^{-3t} - 4y \end{aligned}\right\} \text{1st order}$$

$$\left.\begin{aligned} y y'' + t^2 y &= \cos(t) \\ y'' &= y^2 \end{aligned}\right\} \text{2nd order}$$

highest derivative

partial differential equation (PDE)

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

Normal Form: given function

$$\phi(t, y, y') = 0$$

any 1st order ODE in normal form by solving for the highest derivative Example

$$y' - y - t = 0 \longrightarrow y' = y + t$$

$$y'' - ky' + 3y = 0 \longrightarrow y'' = ky' - 3y$$

to help solve the DE.

Solve for highest order

$$\boxed{y' = f(t, y)}$$

normal forms

Write $t \cancel{+} 4yy' = 0$ in normal form:

Solving y',

$$t + 4yy' = 0$$

$$4yy' = -t$$

$$y' = \frac{-t}{4y}$$

$$\boxed{y' = f(t, y)}$$

<u>Solution of a Differential Equation?</u>

a function that satisfies both sides of the Diff Eq

$$\frac{dy}{dt} = y - t$$

Solution $y(t) = t + 1$

$$\frac{d}{dt}(t+1) = \cancel{(t+1)} - \cancel{t}$$

$$1 = 1$$

Ex. $y(t) = Ce^{-t^2}$ a solution of $\frac{dy}{dt} = -2ty$

C real constant

$$\frac{d}{dt}\left(Ce^{-t^2}\right) = -2tCe^{-t^2}$$

$$C\frac{d}{dt}e^{-t^2} = -2tCe^{-t^2}$$

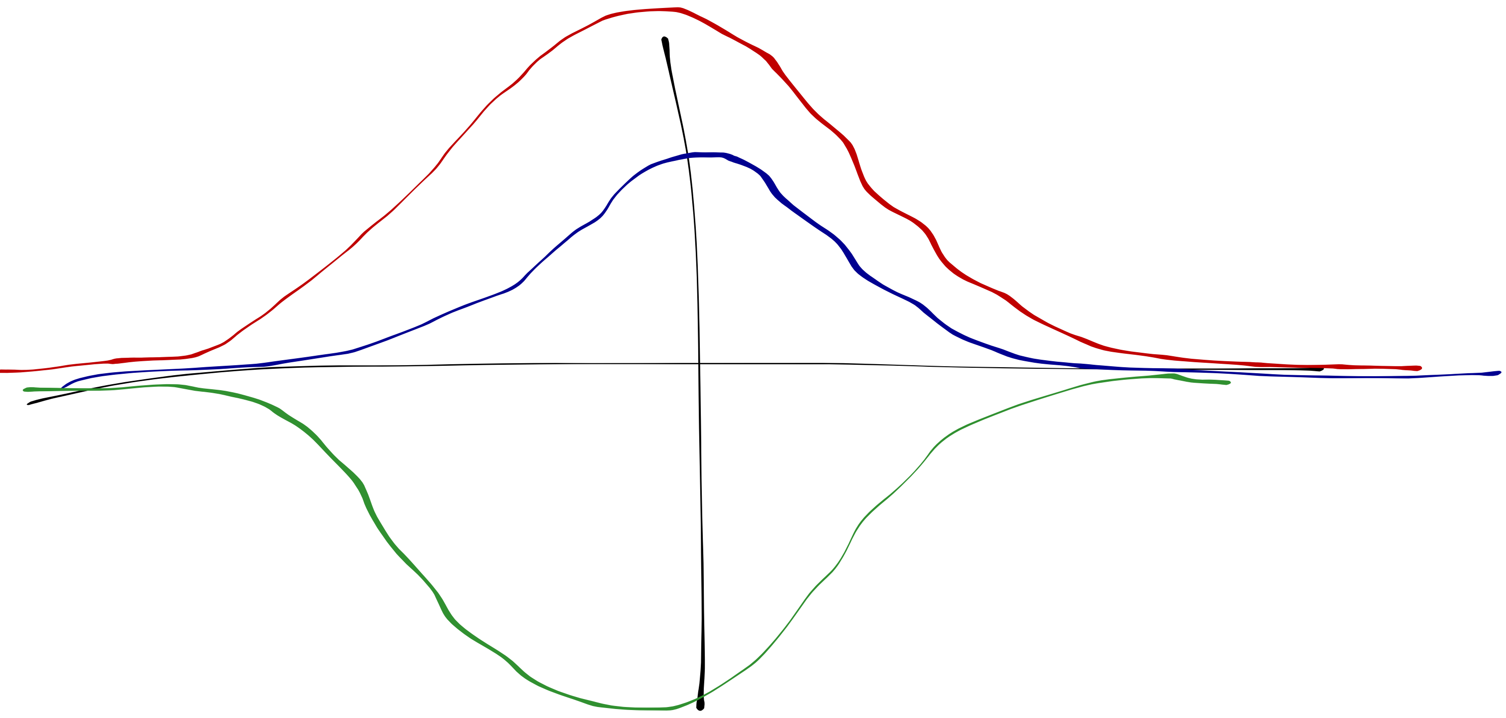
$$C(-2t)e^{-t^2} = -2tCe^{-t^2}$$

$y(t) = Ce^{-t^2}$

$C = 1$

$C = .5$

$C = -1$



General form of a DiffEq

Initial Value Problem

if we have more info, we can get a particular solution

Ex initial population of species

Ex $y(t) = -\dfrac{1}{t-c}$

general solution

$$y' = y^2$$

Find particular solution

$$y(0) = 1$$

Plug $t=0$, $y=1$ into solution, solve for C

$$1 = -\frac{1}{0-C}$$

$$1 = +\frac{1}{+C}$$

$C = 1$ particular solution

$$\boxed{\cancel{y(t) = -\frac{1}{t-c}}}$$

$y(t) = -\frac{1}{t-1}$ $\quad C = \underline{1}$

IVP first order DE
w/ an initial condition

Solution of a IVP

1. $y'(t) = f(t, y(t))$ for all t in ~~on~~ an interval

Contains our initial value

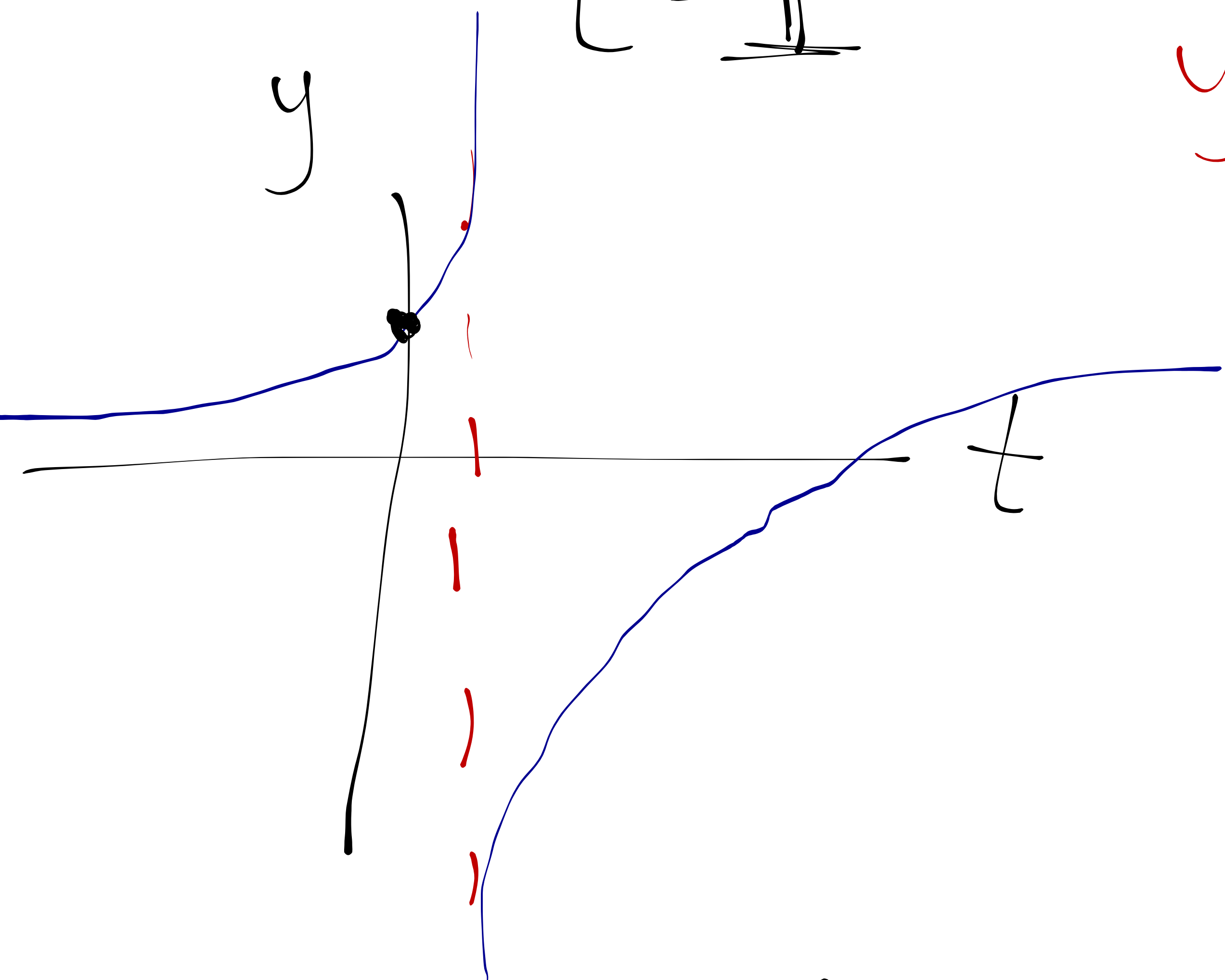
2. $y(t_0) = y_0$

Interval of existence largest interval over which the solution can be defined and remains a solution, solution must be differentiable and continuous

$$y(t) = \frac{-1}{t-1}$$

$t = 1$

$y \to \infty$



maximum interval, contain our intial value $y(0) = 1$

$(0, 1)$, interval of existence

$(-\infty, 1)$

Ex $s = \sqrt{r}$ general solution

$$s(r) = \frac{2}{3} r^{3/2} + C$$

Check solution: plug in

$s' = \sqrt{r}$ $\qquad$ $\underline{s(r) = \frac{2}{3} r^{3/2} + C}$

$$\frac{d}{dr}\left(\frac{2}{3} r^{3/2} + C\right) = \sqrt{r}$$

$$\cancel{\frac{2}{3}}\cancel{\left(\frac{3}{2}\right)} r^{1/2} = \sqrt{r}$$

$$\sqrt{r} = \sqrt{r} \checkmark$$

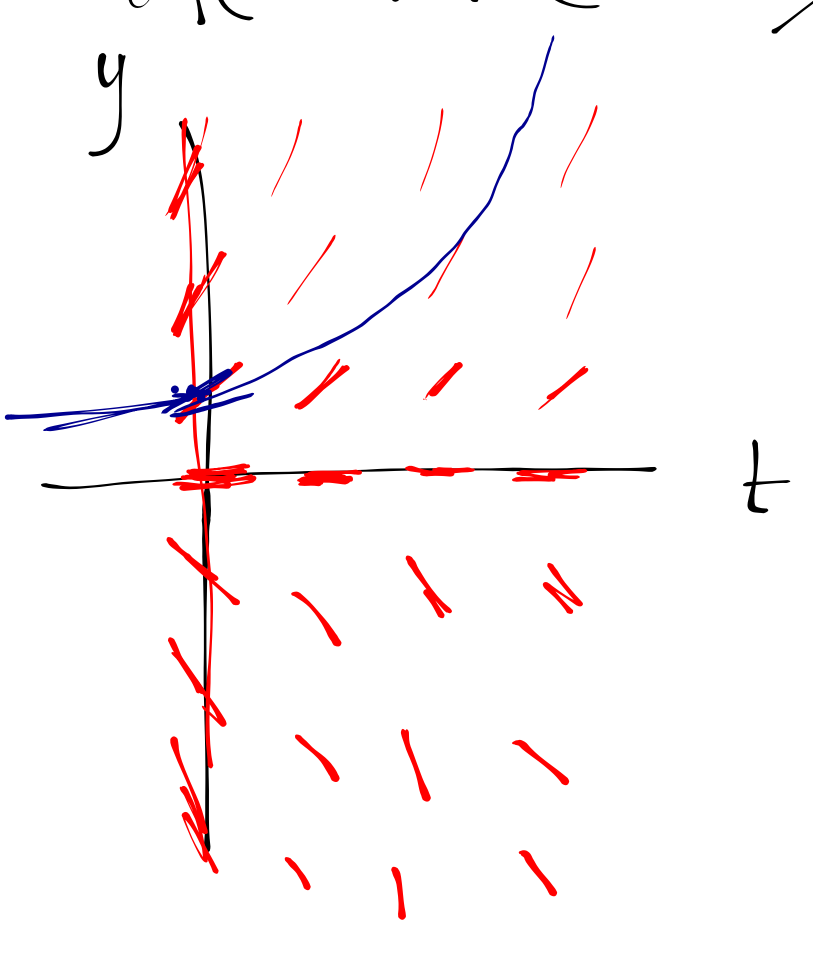
Largest interval that the solution exists $[0, \infty)$

Geometric meaning

$y' = f(t, y)$, solution $y(t)$

at a point (t_0, y_0)

derivate $\longrightarrow$ slope at a point



$$\boxed{y(t) = Ce^{t}}$$

$$\underline{\underline{y' = y}}$$

$y' = 0$

$t = 0, y = 1$