Math 0290 Schedule and Practice Problems (updated 8/19)

This schedule may fluctuate slightly especially as we figure out how to accommodate your safety and health. Currently there are 13 problem sets (including HW 0) due throughout the semester. This may decrease in number as I determine the best way to deal with the homework assignment near midterm 2, but definitely not increase.

Homework for the week is listed in **red** under description of the material covered that week along with the date it is due. These assignments are to be submitted on Canvas Assignments. Lectures are listed by day including problems you can practice from that section to better understand the material (some of these problems are the homework problems for the following week). Then there are learning objectives for the week. They are split up into many smaller items, so it seems like a lot, but many are not time consuming. You can use this list to make sure you understood everything from the week and review for midterms and exams. *Learning objectives that are surrounded by stars won't be tested on the exam but still maybe useful things to know outside of this course. *

Week 1: Introductions to the course

Homework #0 Due Monday August 24 by midnight

Answer questions about Syllabus

- What do you like about the syllabus?
- What would you change about the syllabus?
- What are you excited about in this course?
- What are you nervous about in this course?
- Any additional comments?

- **August 19: Welcome, Introduction to Differential Equations**
  - Practice problems Chapter 1.1
- **August 21: First Order Initial Value Problems**
  - Practice problems Chapter 2.1 #3-6, 10-15, 21-28.

Students should be able to:

1. Explain what a **differential equation** is
   a. Identify what is required for a differential equation
   b. Give examples of some systems that use differential equations
2. Identify a **first order differential** equation
   a. Write an ordinary differential equation (ODE) in **normal form**
   b. Check a given solution for an ODE
   c. Find a **particular solution** for an ODE in an **initial value problem**

Week 2: Intro to Numerics

Homework #1 due Friday August 28 by midnight:

Chapter 1.1 #1, 5, 7, 11
Chapter 2.1 #1, 3, 5, 12, 13, 15

- **August 24: Numerical Methods. Euler's Method**, **Computer tools including Matlab for Differential Equations**
  - 6.1 Practice problems #1-9, 11
  - https://www.ludu.co/course/matlab/useful-matlab-commands
- **August 26: Numerical Methods. Runge-Kutta Methods**
  - 6.2 Practice problems #1-9
- **August 28: Numerical Methods. Numerical Error**
  - 6.3 Practice problems #1-6, 11-13

Students should be able to:

1. Explain what a **numerical solution** is
2. Apply the **Euler's Method** for modeling differential equations with initial value problems (IVP)
   a. Compute a few steps of the Euler's Method for an IVP
   b. Calculate the error from using Euler's Method compared to an exact solution
   c. Explain the difference between **truncation error** and **roundoff error**
   d. *Apply Euler's Method for a system of differential equations*
3. Apply the **second order Runge-Kutta Method** for modeling differential equations
   a. Calculate the error for the second order Runge-Kutta Method
4. Apply the **fourth order Runge-Kutta Method** for modeling differential equations
   a. Calculate the error for the fourth order Runge-Kutta Method
5. *Use Matlab to apply Euler's Method*
6. *Use Matlab to apply the 2nd order Runge-Kutta Method*
7. *Use Matlab to apply the 4th order Runge-Kutta Method*
8. *Plot Matlab results*

Week 3:
Homework #2 due Friday September 4th by midnight:
Chapter 6.1 #3, 5
Chapter 6.2 #23

- **August 31: Separable Equations**
  - 2.2 Practice problems #1-22, 23-29, 33-35
- **September 2: Models of Motion**
  - 2.3 Practice problems #1-10
- **September 4: First Order Linear Equations**
  - 2.4 Practice problems #1-21, 29

Students should be able to:

1. Explain what a **separable equation** is
   a. Rewrite and solve a separable equation

   b. Describe the case when you cannot solve a separable equation
   c. Explain what a **general solution** is
   d. Explain what an **implicitly defined solution** is and the difference between that and an **explicitly defined solution**
2. Solve for an equation of motion using physical models
   a. Apply the scaling variables method to simplify separation of variables
   b. Explain what a **homogeneous linear equation** and an **inhomogeneous equation** are
   c. Explain what the **coefficients** of the equation are
   d. Find the solution to a homogeneous equation
   e. Find the solution to an inhomogeneous equation

Week 4: Linear first-order equations and Intro to Second order equations

Homework #3 due Friday September 11th by midnight:

Chapter 2.2 #3, 5, 9, 33
Chapter 2.3 #9
Chapter 2.4 #5, 15, 19

- **September 7: Mixing Problems**
  - 2.5 Practice problems #1-7, 9, 10
- **September 9: Electrical Circuits**
  - 3.4 Practice problems #1-19
- **September 11: Second Order Equations**
  - 4.1 Practice problems #1-20, 26-30

Students should be able to:

1. Set up differential equation for a **mixing problem**
   a. Identity the **volume rates** and **concentration in a** mixing problem
   b. Solve a mixing problem differential equation
2. Explain the component laws (**Ohm's law, Faraday's law, capacitance law**)
3. Explain **Kirchhoff's voltage law**
4. Explain **Kirchhoff's current law**
   a. Derive a differential equation for a circuit using the component laws and Kirchhoff's laws
   b. Solve a differential equation derived for a circuit using given variables and initial conditions
5. Identify a **second order differential** equation
   a. Explain how a **forcing term** affects a linear differential equation
   b. Derive a second order differential equation for a **spring-mass equilibrium (SME)** system with an external force
      i. Solve for the **spring constant** in the SME system
      ii. Solve the second order differential equation for a SME system when the system is **undamped**
      iii. Solve for the **period** of the SME system

iv. Solve for the **angular frequency** of the SME system
v. Solve for the **numerical frequency** of the SME system
vi. Define what **linear combination** is
vii. Define what **linearly dependent** and **linearly independent** are
viii. Find a **fundamental set of solutions** to a second order differential equation
ix. Determine the **Wronskian** of two solutions of a second order differential equation
x. Explain the independence of two solutions using the result of the Wronskian calculation

<u>Week 5: Second order equations and harmonic motion</u>

Homework #4 due Friday September 18th by midnight:
Chapter 2.5 #5, 9b
Chapter 3.4 #1, 3, 5, 7, 11
Chapter 4.1 #1, 3, 9, 17

- **September 14: Linear Homogeneous Equations with Constant Coefficients**
  - 4.3 Practice problems #1-36
- **September 16: Linear Homogeneous Equations with Constant Coefficients (con't)**
  - 4.3 Practice problems #1-36
- **September 18: Harmonic Motion**
  - 4.4 Practice problems #1-12, 14-16, 18

Students should be able to:

1. Derive the **characteristic equation** of a differential equation
   a. Solve for the **characteristic root** of a differential equation
   b. Determine if the roots are **two distinct real roots**, **two distinct complex roots**, or **one repeated root**
   c. Use the characteristic roots to determine the **general solution** of the differential equation with two distinct real roots
      i. Find a **particular solution** using a set of initial conditions with two distinct real roots
   d. Use the characteristic roots to determine the **general solution** of the differential equation with two distinct complex roots
      i. Find a **particular solution** using a set of initial conditions with two distinct complex roots
   e. Use the characteristic roots to determine the **general solution** of the differential equation with one repeated root
      i. Find a **particular solution** using a set of initial conditions with one repeated root
2. Identify the equation for **simple harmonic motion (SHM)**
   a. Derive the characteristic equation for SHM
   b. Find the characteristic roots for SHM

c. Solve for the solution for SHM
d. Plot a particular solution for the SHM case
e. Rewrite the solution to the SHM case using **polar form**
f. Identify the **amplitude** of oscillations for a SHM
g. Identify the **phase** of oscillations for a SHM

3. Identify the equation for **damped harmonic motion (DHM)**
    a. Derive the characteristic equation for DHM
    b. Find the characteristic roots for DHM
    c. Solve for the solution for DHM
    d. Plot a particular solution for the DHM case
    e. Rewrite the solution to the DHM case using **polar form**
    f. Identify the **amplitude** of oscillations for a DHM
    g. Identify the **phase** of oscillations for a DHM

Week 6: Inhomogeneous second order equations

Homework #5 due Friday September 25th by midnight:
Chapter 4.3 #1, 9, 17, 35
Chapter 4.4 # 1, 7

- **September 21: Inhomogeneous Equations. Undetermined Coefficients**
    - 4.5 Practice problems #1-29
- **September 23: Undetermined Coefficients (continued)**
    - 4.5 Practice problems #1-29
- **September 25: Inhomogeneous Equations. Variation of Parameters**
    - 4.6 Practice problems #1-10

Students should be able to:

1. Identify the form of an **inhomogeneous linear equation**
    a. Find the general solution to the inhomogeneous equation
    b. Find the particular solution to the inhomogeneous equation
2. Explain the **method of undetermined coefficients**
    a. Identify the case when the **method of undetermined coefficients** can be applied to differential equation
    b. Apply the method of undetermined coefficients to the case with **exponential forcing terms**
    c. Apply the method of undetermined coefficients to the case with **trigonometric forcing terms**
        i. Apply the **complex method** to the case with trigonometric forcing terms
    d. Apply the method of undetermined coefficients to the case of **polynomial forcing terms**
    e. Identify exceptional cases when the method of undetermined coefficient cannot be used

    f. Apply the method of undetermined coefficients to the case of a linear combination of forcing terms (exponential, trigonometric, polynomial)
3. Apply the technique of **variation of parameters** to find a particular solution to a second order differential equation

<u>Week 7: Forced harmonic Motion, Laplace Transforms</u>

Midterm #1 will be available from Tuesday, September 29th 12:00pm to Thursday October 1st 11:59am, covering material up to and including HW4

- **September 28: Forced Harmonic Motion**
    - 4.7 Practice problems #3-11
- **September 30: Midterm 1, no class**
- **October 2: Laplace Transform**
    - 5.1 Practice problems #1-29

Students should be able to:

1. Apply the technique of undetermined coefficients to analyze **harmonic motion with an external sinusoidal forcing term with no damping**
    a. Explain what the **driving frequency** of an equation of harmonic motion with external sinusoidal forcing is
    b. Determine how the driving frequency compares to the natural frequency
    c. Find the particular solution when the **driving frequency does not equal the natural frequency**
        i. Find the general solution to the inhomogeneous equation
        ii. Plot the solution to harmonic motion with an external sinusoidal forcing term
        iii. Explain what **beats** are
        iv. Calculate the mean frequency and half difference
    d. Find the particular solution when the **driving frequency equals the natural frequency**
        i. Find the general solution to the inhomogeneous equation
        ii. Explain what **resonance** is
2. Apply the technique of undetermined coefficients to analyze **harmonic motion with an external sinusoidal forcing term with damping**
    a. Determine the **transfer function**
    b. Determine the **gain**
    c. Determine the **transient term** and the **steady state term**
3. Define a **Laplace Transform** and state why it is useful
    a. Apply a Laplace transform to an exponential function
    b. Apply a Laplace transform to a linear function
    c. Apply a Laplace transform to a sinusoidal function
    d. Apply a Laplace transform to piecewise continuous functions

<u>Week 8: Beginning Laplace Transforms</u>

Homework #6 due Friday October 9th by midnight:

Chapter 4.5 #1, 5, 11, 15, 19
Chapter 4.6 #1, 3, 5
Chapter 4.7 #3, 11

- **October 5: Laplace Transform. Basic Properties**
  - 5.2 Practice problems #1-41
- **October 7: The Inverse Laplace Transform**
  - 5.3 Practice problems #1-36
- **October 9: Using the Laplace Transform to Solve Des**
  - 5.4 Practice problems #1-26

Students should be able to:

1. Define a **Laplace transform** on derivatives
   a. Apply a Laplace transform on piecewise differentiable and continuous functions
2. Recognize and apply properties of the Laplace transform
   a. Apply the linear property of a Laplace transform
   b. Apply the exponential property of a Laplace transform
   c. Apply the derivative property of a Laplace transform
3. Find the inverse Laplace transform
   a. Compute inverse Laplace transforms using a table
   b. Compute the partial fraction decomposition for a rational function
      i. By applying the coefficient method
      ii. By apply the substitution method
   c. Compute the inverse Laplace transform of rational functions
4. Use the Laplace transform to solve differential equations
   a. Use the Laplace transform to solve the initial value problem for a homogeneous differential equation
   b. Use the Laplace transform to solve the initial value problem for a inhomogeneous differential equation
   c. Use the Laplace transform to solve a higher order equation

<u>Week 9:</u>

Homework #7 due Friday October 16th by midnight:
Chapter 5.1 #7, 13, 15, 29
Chapter 5.2 #5, 11, 19, 29
Chapter 5.3 #3, 7, 11, 19

- **October 12: Discontinuous Forcing Term**
    - 5.5 Practice problems #1-25
- **October 14: Student Self-Care Day (no classes)**
- **October 16: The Dirac Delta Function**
    - 5.6 Practice problems #1-9

Students should be able to:

1. Define the Heaviside function
    a. Rewrite piecewise differentiable function in terms of the Heaviside function
2. Use the Heaviside function to find the Laplace transform of a piecewise differential function
3. Use the Heaviside function to find inverse Laplace transform of a function
4. Solve an initial value problem with piecewise defined forcing function
5. Compute the Laplace transform of the periodic function the square wave
6. Define what impulse is in terms of physical concepts
7. Define the delta function
    a. Calculate the Laplace transform of a delta function
8. Find the unit impulse response function for a differential equation

<u>Week 10: Laplace Transform (cont), Systems of differential equations</u>

Homework #8 due Friday October 23rd by midnight:
Chapter 5.4 #7, 11, 21
Chapter 5.5 #1, 3, 11, 17
Chapter 5.6 #2, 3, 5, 7

- **October 19: Convolutions**
    - 5.7 Practice problems #4-24
- **October 21: Introduction to Systems**
    - 8.1 Practice problems #1-16
- **October 23: Systems (cont)**
    - 8.2 Practice problems #1-6, 13-16

Students should be able to:

1. Define a convolution of two functions
    a. Calculate the Laplace transform of a convolution
    b. Find a solution to the general initial value problem using the Laplace transform of a convolution
    c. Apply the property of a convolution of a piecewise continuous function and a delta function
    d. Apply the property of a derivative of a convolution of two piecewise continuous functions

2. Identify what makes a model **nonlinear**
3. Identify what makes a model **autonomous**
4. Identify the **susceptible-infected-recovered (SIR)** model
5. Write a system of equations in **vector notation**
    a. Determine the **dimension** of a system of first-order equations
    b. Define a planar system
    c. Reduce a higher-order equation to a system of first order equations
6. Identify the **predator-prey/Lotka-Volterra** system
    a. Plot the solutions to the Lokta-Volterra system dependent on time
    b. Plot the **parametric curves** of the solution of the Lotka-Volterra model
    c. Define **phase plane,** and **phase plane plot/solution curve**
    d. Identify the **phase space** of a system of n-dimensions
    e. Plot **the direction field** of a planar system

Week 11: Systems of differential equations, Constant coefficients

Homework #9 due Friday October 30th by midnight:
Chapter 5.7 #6, 8, 10
Chapter 8.1 #5, 7, 13, 15
Chapter 8.2 #11, 13, 15 (use pplane.jar)

- **October 26: Systems (cont)**
    - 8.3 Practice problems #1-6
- **October 28: Linear Systems with Constant Coefficients**
    - 9.1 Practice problems #1-8, 16-23
- **October 30: Planar Systems**
    - 9.2 Practice problems #1-27, 58-61

Students should be able to:

1. Show an IVP has a **unique**, **well-defined** solution
    a. Explain why two solution curves in phase space for an autonomous system cannot meet at a point unless the curves coincide
2. Explain what a nullcline of a system of autonomous equations is
    a. Find the **nullclines** of a system of autonomous equations
    b. Plot the nullclines of a system of autonomous equations
    c. Find an **equilibrium point** of a system of autonomous equations
    d. Determine the **equilibrium solution**
    e. Explain what an **equilibrium solution** is
3. Explain what an **eigenvalue** of a matrix means
4. Explain what an **eigenvector** of a matrix is
5. Find the eigenvalues and eigenvectors of a matrix
6. Use eigenvalues and eigenvectors to find a fundamental set of solutions to a system of differential equations
7. Determine that solutions of planar systems are linearly independent
8. Use real eigenvalues and eigenvectors to find a general solution of a planar system

9. Use complex eigenvalues and eigenvectors to find a general solution of a planar system
10. Use one real eigenvalue with multiplicity of 2 to find a general solution of a planar system

Week 12: Nonlinear Systems

Midterm #2 will be available from Tuesday, November 3rd 12:00pm to Thursday November 5th 11:59am covering material up to and including HW8

Homework #10 due Friday November 6th by midnight (to be changed)

Chapter 8.3 #1, 3, 5
Chapter 9.1 #3, 5, 17, 19
Chapter 9.2 #13, 13, 15, 59

- **November 2: Phase Plane Portraits**
  - 9.3 Practice problems #1-23
- **November 4: Midterm #2, no class**
- **November 6: Nonlinear Systems: Equilibria, Linearization**
  - 10.1 Practice problems #1-16

Students should be able to:

1. Find the three types of solutions when there are two real distinct eigenvalues
   a. Find the **exponential solutions/half-life solutions**
      i. Determine what eigenvalues make the solution exponential
      ii. Determine what makes a solution **unstable**
      iii. Determine what makes a solution **stable**
   b. Find **saddle point** solutions
      i. Determine what eigenvalues makes the solution a saddle point
      ii. Explain what a saddle point of a planar system is
      iii. Explain what a separatrix is
      iv. Find the separatrix of a planar system
   c. Find the **nodal sink/nodal source** solutions
      i. Determine what eigenvalues make the solution nodal source/nodal sink
      ii. Explain what solutions look like
2. Find the three types of solutions when the eigenvalues are complex
   a. Explain what solutions do when there is a **center**
      i. Plot solution curves
      ii. Find the general solution
   b. Explain what solutions do when there is a **spiral sink**
      i. Plot solution curves

        ii. Find the general solution
        iii. Determine the **direction of rotation** of solutions
    c. Explain what solutions do when there is a **spiral source**
        i. Plot solution curves
        ii. Find the general solution
        iii. Determine if the **direction of rotation** of solutions
3. Explain what makes a differential equation **nonlinear**
    a. Find equilibrium points of a nonlinear differential equation
    b. Linearize the equation at the equilibrium point
    c. Characterize equilibrium points (saddle point, nodal sink, nodal source, spiral sink, spiral source)

<u>Week 13: Fourier Series</u>

Homework #11 due Friday November 13th by midnight:
Chapter 9.3 #21
Chapter 10.1 #3, 7, 15

- **November 9: Fourier Series**
    - 12.1 Practice problems #1-22
- **November 11: Fourier Cosine and Sine Series**
    - 12.3 Practice problems #1-32
- **November 13:**
    - 12.4 Practice problems #1-11

Students should be able to:

1. Show that terms of the Fourier series are **orthogonal**
2. Find coefficients of terms in the Fourier series
3. Define piecewise continuous
4. Determine a Fourier series for a corresponding function
5. Determine if the Fourier series converge
6. Determine if a function is odd or even
7. Determine the Fourier series of an even function
8. Determine the Fourier series of an odd function
9. Use Euler's formula to rewrite a real Fourier series in complex form
10. Determine the complex Fourier series from a corresponding function

<u>Week 14:</u>

Homework #12 due Friday November 20th by midnight:
Chapter 12.1 #5, 7, 13, 17

Chapter 12.3 #3, 7, 19, 31
Chapter 12.4 #3

- **November 16: Separation of Variables**
  - 13.2 Practice problems #1-18
- **November 18: Review**
- **November 20: Review**

Students should be able to:

1. Solve the IVB for the **heat equation**
   a. Explain what makes the boundary conditions **homogeneous**
   b. Use separation of variables to rewrite a **partial differential** equation as two ODE
   c. Set up the **Strum-Louiville** problem
   d. Explain what an **eigenfuction i**s
   e. Satisfy initial conditions of SL problem
   f. Use a Fourier series to solve the SL problem
   g. Find a **steady-state temperature**

Reading Day: Saturday November 21st

Final Exam:

Thanksgiving!

Students should be able to: REST! Enjoy the holidays! We made it!