MATH 0290 SEC 1050 Introduction to Differential Equations
HW #4 Due Friday September 18th 11:59pm EST

Questions from Polking, Boggess and Arnold, *Differential Equations with Boundary Value Problems*, second edition

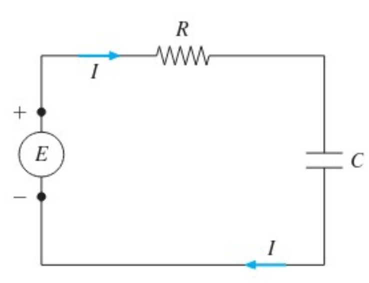
Chapter 2.5 #5, 9b
Chapter 3.4 #1, 3, 5, 7, 11
Chapter 4.1 #1, 3, 9, 17

Chapter 2.5
5. A 50-gal tank initially contains 20 gal of pure water. Salt-water solution containing 0.5 lb of salt for each gallon of water begins entering the tank at a rate of 4 gal/min. Simultaneously, a drain is opened at the bottom of the tank, allowing the salt-water solution to leave the tank at a rate of 2 gal/min. What is the salt content (lb) in the tank at the precise moment that the tank is full of salt-water solution?

9b. A lake, with volume $V = 100\ km^3$, is fed by a river at a rate of $r\ km^3/yr$. In addition, there is a factory on the lake that introduces a pollutant into the lake at the rate of $p\ km^3/yr$. There is another river that is fed by the lake at a rate that keeps the volume of the lake constant. This means that the rate of flow from the lake into the outlet river is $(p + r) km^3/yr$. Let $x(t)$ denote the volume of the pollutant in the lake at time t. Then $c(t) = x(t)/V$ is the concentration of the pollutant.

It has been determined that a concentration of over 2% is hazardous for the fish in the lake. Suppose that $r = 50\ km^3/yr$, $p = 2km^3/yr$, and the initial concentration of pollutant in the lake is zero. How long willit take the lake to become hazardous to the health ofthe fish?

Chapter 3.4
A resistor (20 Ω) and capacitor (0.1 F) are joined in series with an electromotive force (emf) $E = E(t)$, as shown in the figure below. If there is no charge on the capacitor at time $t = 0$, find the ensuing charge on the capacitor at time t for the given emf in exercises 1, 3, and 5

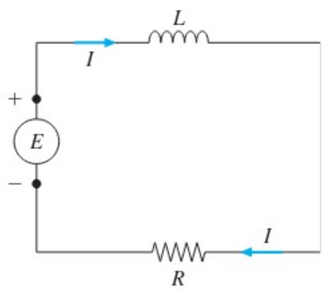


1. $E(t) = 100$ V

3. $E(t) = 100 \sin 2t$ V

5. $E(t) = 100 - t$ V

An inductor (1 H) and resistor (0.1 Ω) are joined in series with an electromotive force (emf) $E = E(t)$, as shown in the figure below. If there is no current in the circuit at time $t = 0$, find the ensuing current in the circuit at time t for the given emf in exercises 7 and 11.



7. $E(t) = 1\text{V}$

11. $E(t) = 10 - 2t$ V

Chapter 4.1

For each of the second-order differential equations in exercises 1 and 3, decide whether the equation is linear or nonlinear. If the equation is linear, state whether the equation is homogeneous or inhomogeneous.

1. $y'' + 3y' + 5y = 3\cos 2t$

3. $t^2y'' + (1 - y)y' = \cos 2t$

9. In an experiment, 2-kg mass is suspended from a spring. The displacement of the spring-mass equilibrium is measured to be 50 cm. If the mass is then displaced 12 cm downward from its spring-mass equilibrium and released from rest, set up (but do not solve) the initial value problem that models this experiment. Assume no damping is present.

17. Use Definition 1.22 to explain why $y_1(t)$ and $y_2(t)$ are linearly independent solutions of the giving differential equation. In addition, calculate the Wronskian and use it to explain the independence of the giving solutions

$y'' - y' - 2y = 0$, $y_1(t) = e^{-t}$, $y_2(t) = e^{2t}$

Definition 1.22: Two functions u and v are said to be ***linearly independent*** on the interval (α, β) if neither is a constant multiple of the other on the interval. If one is constant multiple of the other on (α, β), they are said to be ***linearly dependent*** there.