Differential Equations 9/2/2020

HW #2 due Friday @midnight EST

Office Hours tomorrow @4-5pm EST

Add/Drop deadline Sept 4 ←

Please do Poll

$$y' = \frac{h(t)}{f(y)} \quad . \quad y' = \boxed{g(t)}\, h(y)$$

$$y' = ty \qquad \frac{dy}{y} = t\,dt$$

$$y' = \frac{t}{y} \qquad y\,dy = t\,dt$$

$$y' = (3-y)\,\boxed{t^0}^{=1} \qquad \frac{dy}{3-y} = dt$$

$$y' = y\cos(t) \qquad \frac{dy}{y} = \cos t\,dt$$

$$\boxed{y' = t - y \quad \text{can't use separation of variables}}$$

Implicitly defined function:

$$y' = \frac{e^x}{1+y} \qquad y(0) = 1$$
$$y(0) = -4$$

$$\int dy\,(1+y) = \int dx\, e^x$$

$$y^2 + 2y - 2(e^x + C) = 0$$

$$y = \frac{-2 \pm \sqrt{2^2 - 4(-2)(e^x + C)}}{2}$$

$$y = -1 \pm \sqrt{1 + 2(e^x + C)}$$

$y(0) = 1$

$$y = -1 + \sqrt{1 + 2(e^x + C)}$$
$$1 = -1 + \sqrt{1 + 2(\cancel{e^0} + C)}$$

$$1 = -1 + \sqrt{1 + 2(1 + c)}$$

$$2 = \sqrt{1 + 2 + 2c}$$

$$2 = \sqrt{3 + 2c}$$

$$4 = 3 + 2c$$

$$\boxed{\frac{1}{2} = c}$$

$$y(0) = 1$$

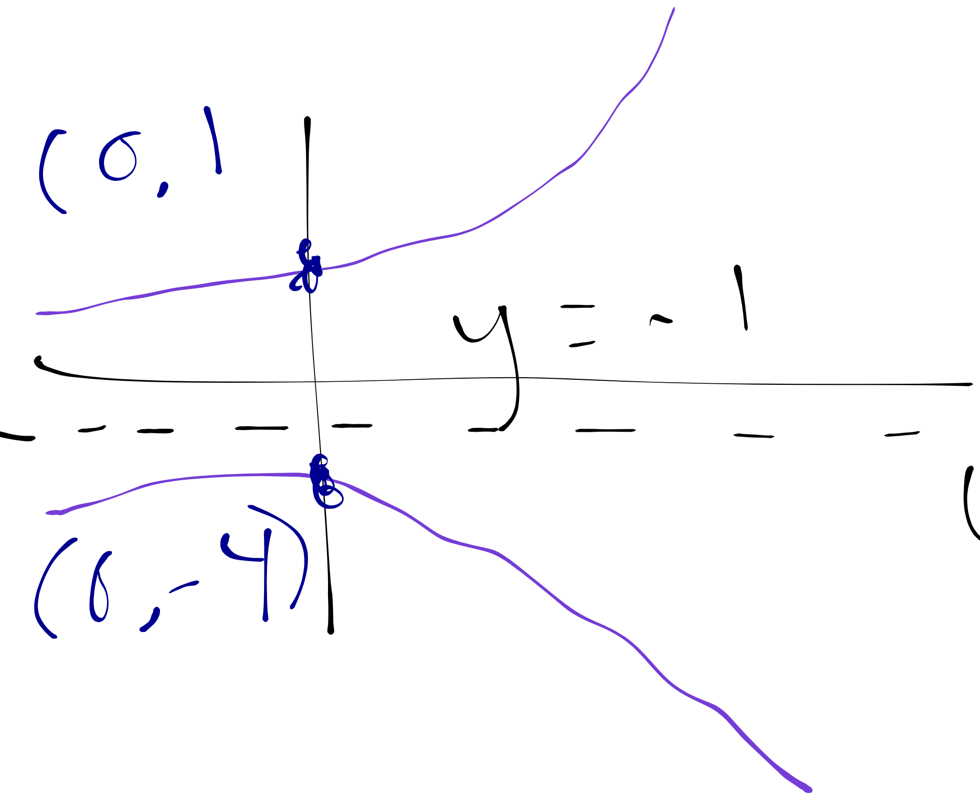
$$\boxed{y(x) = -1 + \sqrt{2 + 2e^{x}}}$$

$$y(0) = -4$$

$$-4 = -1 - \sqrt{1 + 2(e^{0} + c)}$$

$$\boxed{3 = c}$$

$$\boxed{y(x) = -1 - \sqrt{7 + 2e^{x}}}$$



$$y(x) = -1 + \sqrt{2+2e^x}$$

$$y(x) = -1 - \sqrt{7+2e^x}$$

$$y' = \frac{e^x}{1+y}$$

$y = -1$

okay

Interval of Existence

$(-\infty, \infty)$

Ex 2.2 #21

$y' = 2 - y$ $\qquad$ $y(0) = 3$

$\qquad\qquad\qquad\quad\;\, y(0) = 1$

$$\frac{dy}{dt} = 2 - y$$

$$\int \frac{dy}{2-y} = \int dt$$

$$-\ln|2-y| = t + C$$

$$|2-y| = e^{-t-c}$$

$$|2-y| = Ae^{-t}$$

$+(2-y)$ $\qquad\qquad$ $-(2-y)$

$+(2-y) = Ae^{-t}$

$\boxed{2 - Ae^{-t} = y}$

$y(0) = 3$

$2 - Ae^{0} = 3$

$-A = 1$

$A = -1$

$2-(-1)e^{-t} = y$

$\boxed{2+e^{-t} = y}$ ~~t~~

$-(2-y) = Ae^{-t}$

$-2+y = Ae^{-t}$

$\boxed{y = Ae^{-t}+2}$

$y(0) = 3$

$3 = Ae^{0}+2$

$1 = A$

$\boxed{y = e^{-t}+2}$

$y(0) = 1$

$y = 2+Ae^{-t}$

$1 = 2+Ae^{0}$

$-1 = A$

$y(x) = 2 - e^{-t}$

Models of Motion

$F = ma$

free fall

$$\cancel{m}\frac{d^2x}{dt^2} = -\cancel{m}g$$

$$\frac{d^2x}{dt^2} = -g \qquad \frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -g$$

$$\int_{v_0}^{v} dv = -\int_{t_0}^{t} g\,dt \qquad t_0 = 0$$

$$v - v_0 = -g(t - \cancel{t_0})$$

$$v = v_0 - gt \qquad v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = v_0 - gt$$

$$\int_{x_0}^{x} dx = \int_0^t (v_0 - gt)\,dt$$

$$x - x_0 = v_0 t - \frac{g}{2}t^2$$

$$\boxed{x = x_0 + v_0 t - \frac{g}{2}t^2}$$

Air Resistance

$v = 0 \quad F_{ar} = 0$

$v = v_0 \quad F$ opposite direction of motion

$$R(x, v) = -r(x, v)\, v$$

$$F = -mg + R(v)$$

$$= -mg - rv$$

$$\cancel{m}\frac{dv}{dt} = -\cancel{m}g - \frac{rv}{m}$$

$$\int \frac{dv}{-\left(g + \frac{rv}{m}\right)} = \int dt$$

$$-\frac{m}{r} \ln\left(g + \frac{rv}{m}\right) = t + C$$

$$\frac{m}{r}\ln\left(g+\frac{rv}{m}\right) = -t + C$$

$$g+\frac{rv}{m} = e^{-\frac{r}{m}t+\overbrace{\frac{r}{m}C}^{B}}$$

$$\frac{rv}{m} = e^{-\frac{r}{m}t}\,\overbrace{e^{B}}^{A} - g$$

$$v = \frac{m}{r}Ae^{-\frac{r}{m}t} - \frac{gm}{r}$$

what happens $t \to \infty$

$$v = \cancelto{0}{\frac{m}{r}Ae^{-\infty}} - \frac{gm}{r}$$

$$V_{term} = -\frac{mg}{r}$$

$$\frac{dx}{dt} = \frac{mC}{r} e^{-tr/m} - g\frac{m}{r}$$

$$\int dx = \int \left(\quad \right) dt$$

$$\boxed{x(t) = -\frac{m^2 C}{r^2} e^{-tr/m} - g\frac{m}{r} t + D}$$

C_2