MATH 0290 SEC 1050 Introduction to Differential Equations
HW# 1 Due Friday August 28^{th} 11:59pm

UPDATED 8/25 2pm Fixed typos in 1.1 #5 and 2.1 #5

Questions from Polking, Boggess and Arnold, *Differential Equations with Boundary Value Problems*, second edition

Chapter 1.1 #1, 5, 7, 11
Chapter 2.1 #1, 3, 5, 12, 13, 15

Chapter 1.1

The phrase "y is proportional to x" implies that y is related to x via the equation $y = kx$, where k is a constant. In a similar manner, "y is proportional to the square of x" implies $y = kx^2$, "y is proportional to the product of x and z" implies $y = kxz$, and "y is inversely proportional to the cube of x" implies $y = k/x^3$. For example, when Newton proposed that the force of attraction of one body on another is proportional to the produce of the masses and inversely proportional to the square of the distance between them, we can immediately write

$$F = \frac{GMm}{r^2}$$

where G is the constant of proportionality, usually known as the universal gravitational constant. In Exercises 1, 5, 7 and 11, use these ideas to model each application with a different equation. All rates are assumed to be with respect to time.

1. The rate of growth of bacteria in a petri dish is proportional to the number of bacteria in the dish.
3. A certain area can sustain a maximum population of 100 ferrets. The rate of growth of a population of ferrets in this area is proportional to the product of the population and the difference between the actual population and the maximum sustainable population
5. The rate of decay of a certain substance is inversely proportional to the amount of substance remaining
7. A thermometer is placed in a glass of ice water and allowed to cool for an extended period of time. The thermometer is removed from the ice water and placed in a room having temperature 77 degrees F. The rate at which the thermometer warms is proportional to the difference in the room temperature and the temperature of the thermometer.
11. The voltage drop across an inductor is proportional to the rate at which the current is changing with respect to time.

Chapter 2.1

1. Given the function $\phi(x, y, z) = x^2z + (1 + x)y$, place the ordinary differential equation $\phi(t, y, y') = 0$ in normal form.

For exercises 3 and 5, Show that the given solution is a general solution of the differential equation. Use a computer or calculator to sketch the solution for the given values of the arbitrary/ constant. Experiment with different intervals for t until you have a plot that shows what you consider to be the most important behavior of the family.

3. $y' = -ty$, $\quad y(t) = Ce^{-\frac{t^2}{2}}$, $\quad C = -3, -2, \ldots, 3$

5. $y' + \left(\frac{1}{2}\right) y = 2\cos t$, $\quad y(t) = \left(\frac{4}{5}\right)\cos t + \left(\frac{8}{5}\right)\sin t + Ce^{-\frac{t}{2}}$, $\quad C = -5, -4, \ldots, 5$

In exercises 12, 13 and 15, use the given general solution to find a solution of the differential equation having the given initial condition. Sketch the solution, the initial condition, and discuss the solution's interval of existence.

12. $y' + 4y = \cos t$, $\quad y(t) = \frac{4}{17}\cos t + \frac{1}{17}\sin t + Ce^{-4t}$, $\quad y(0) = -1$

13. $ty' + y = t^2$, $\quad y(t) = \frac{1}{3}t^2 + \frac{C}{t}$, $\quad y(1) = 2$

15. $y' = y(2 + y)$, $\quad y(t) = 2/(-1 + Ce^{-2t})$, $\quad y(0) = -3$