Differential Equations 8/28

HW #1 Due tonight @ midnight EST

HW #2 is up

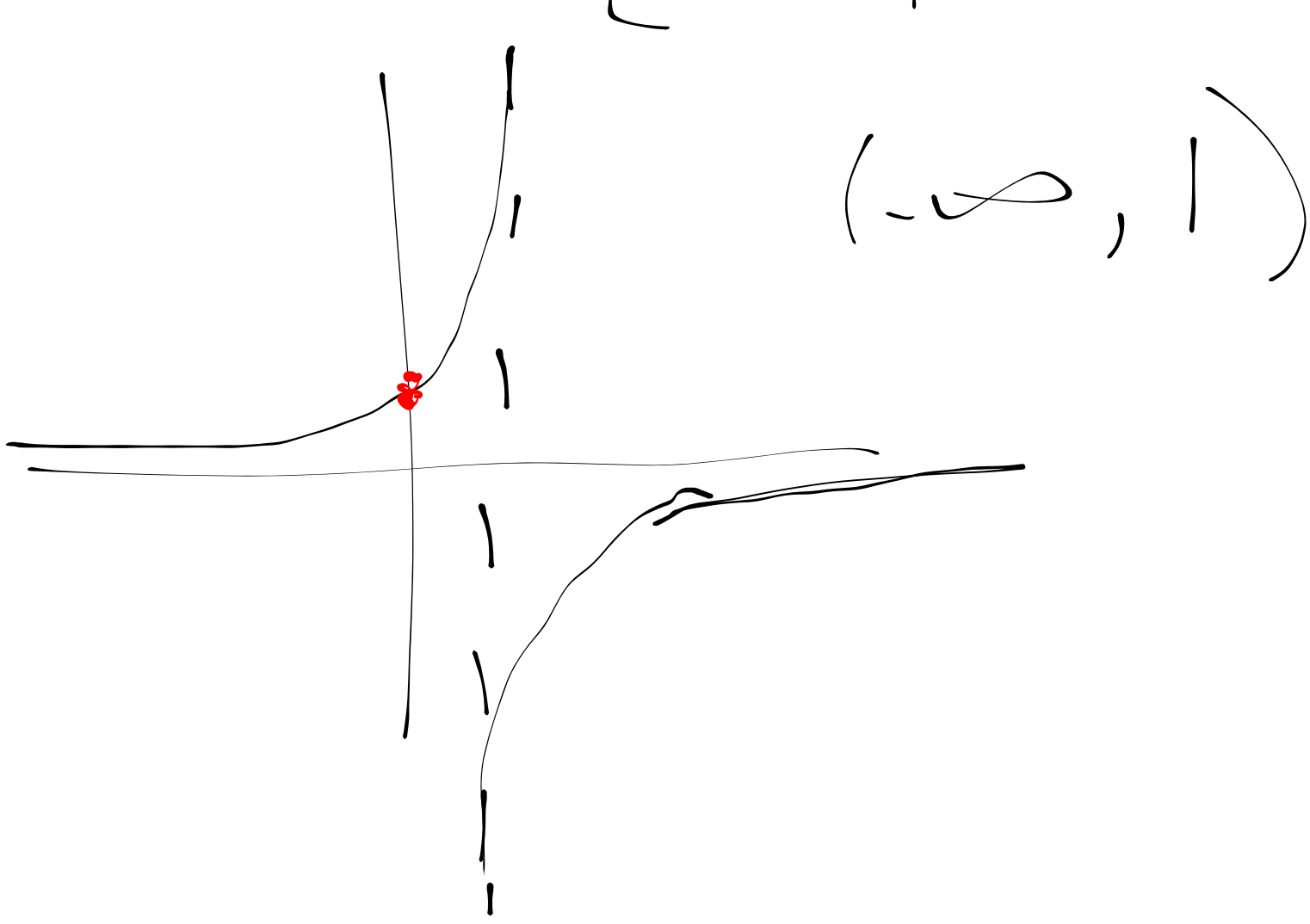
Matlab codes on Canvas

Please do the poll

$$y(t) = \frac{-1}{t-1} \qquad y(0) = 1$$

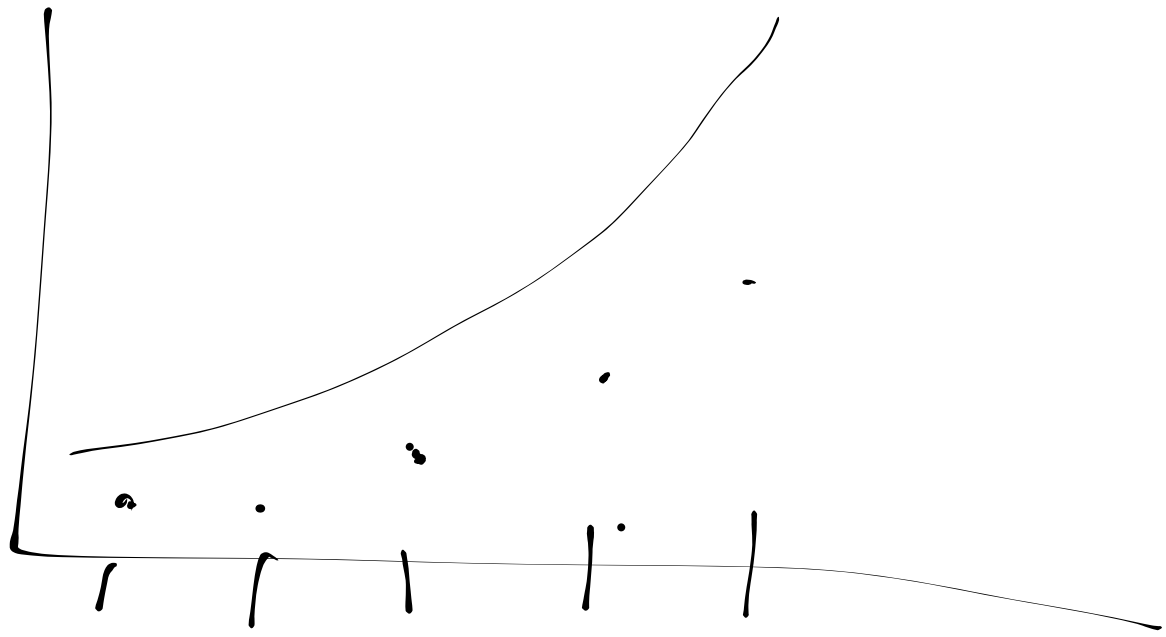
$t - 1 = 0$

$t = 1$



Runge-Kutta Methods

fixed step solver

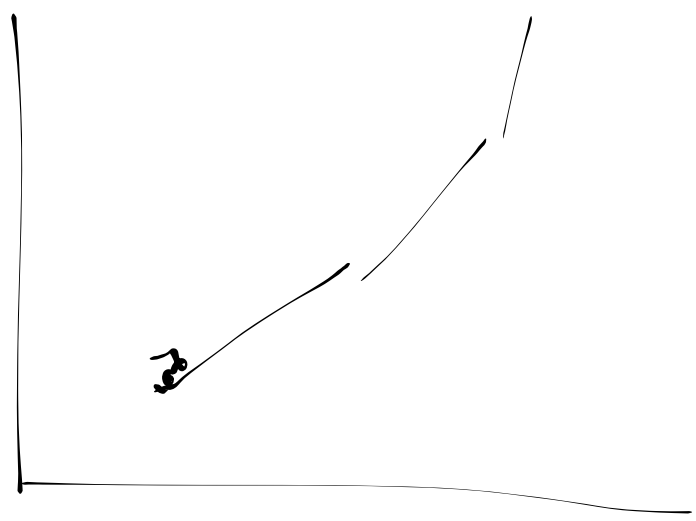


h h h h

Euler's method

$$y_k = y_{k-1} + f(t_k, y_k)h$$

$$y' = f(t_k, y_k)$$



Start (t_0, y_0)

$$\begin{cases} s_1 = f(t_0, y_0) \\ s_2 = f(t_0 + h, y_0 + h s_1) \end{cases}$$

$$y_1 = y_0 + \frac{s_1 + s_2}{2} h$$

average of two slopes

$$t_1 = t_0 + h$$

For step K

$$s_1 = f(t_{k-1}, y_{k-1})$$

$$s_2 = f(t_{k-1} + h, y_{k-1} + h s_1)$$

$$y_k = y_{k-1} + h \frac{s_1 + s_2}{2}$$

$$t_k = t_{k-1} + h$$

Euler Error

$$\text{Maximum error} \leq \frac{M}{L}\left(e^{L(b-a)}\right) h$$

$0 < h <$

Runge Kutta

$$\text{Maximum error} \leq \frac{M}{L}\left(e^{L(b-a)}\right) h^2$$

Smaller than Euler

$$y' = f(t,y) = y - t$$

$$y(1) = 1 \qquad h = 0.1$$

$$t_0 = 1 \qquad y_0 = 1$$

$$s_1 = f(t_0, y_0)$$

$$= y_0 - t_0$$

$$= 1 - 1$$

$$= 0$$

$$s_2 = f(t_0 + h, y_0 + h s_1)$$

$$= y_0 + h s_1 - (t_0 + h)$$

$$= 1 - 1.1$$

$$= -0.1$$

$$y_1 = y_0 + h\frac{s_1 + s_2}{2}$$

$$= 1 + 0.1\left(\frac{0 - 0.1}{2}\right)$$

$$\boxed{y_1 = 0.995}$$

$$t_1 = t_0 + h$$

$$= 1 + 0.1$$

$$\boxed{t_1 = 1.1}$$

Fourth order Runge kutta

$$s_1 = f(t_0, y_0)$$

$$s_2 = f\left(t_0 + \frac{h}{2}, y_0 + \frac{h}{2} s_1\right)$$

$$s_3 = f\left(t_0 + \frac{h}{2}, y_0 + \frac{h}{2} s_2\right)$$

$$s_4 = f(t_0 + h, y_0 + h s_3)$$

weighted average

$$y_1 = y_0 + h \frac{s_1 + 2s_2 + 2s_3 + s_4}{6}$$

taylor expansion

$$\text{Maximum error} \leq \frac{M}{L}\left(e^{L(b-a)} - 1\right)h^4$$