Differential Equations 9/9/2020

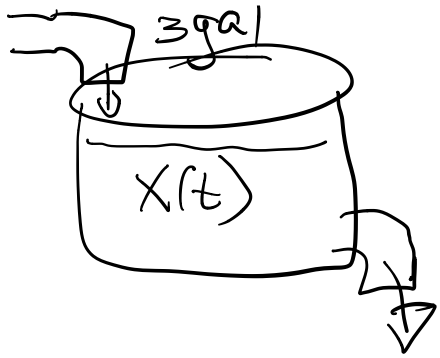
HW # 3 due Friday @midnight EST

Office Hours tomorrow 4-5pm

*Practice Exams on Canvas → Files → Practice Exams

Extra Credit opportunies coming soon!

Midterm #1 Sept 29 - Oct 1 12pm EST



X(t) salt in solution

600 gal tank with 300 gals pure water

in: 1.5 lbs salt/gal

total Volume increase 2 gal/min

$V(t) = 300 + 2t$

rate in: 4.5 lbs/min

rate out: $\frac{X(t)}{300+2t}$ lb/min

$$\frac{dx}{dt} = 4.5 - \frac{X(t)}{300+2t}$$

$$X_n = \frac{1}{\sqrt{300+2t}}$$

$\boxed{X = V(t)\, X_n}$ plug into $\frac{dx}{dt}$

Solve for r

$$\frac{dV}{dt} = 4.5\sqrt{300+2t}$$

$$\int dV = \int 4.5\sqrt{300+2t}$$

$$V = 4.5(300+2t)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{\cancel{2}} + C$$

$$V = \frac{4.5}{3}(300+2t)^{3/2} + C$$

$$X = V X_h$$

$$X = \left(\frac{4.5}{3}(300+2t)^{3/2} + C\right)\frac{1}{(300+2t)^{1/2}}$$

$$= \frac{4.5}{3}(300+2t) + \frac{C}{(300+2t)^{1/2}}$$

$$\boxed{X(t) = 450 + 3t + \frac{C}{\sqrt{300+2t}}}$$

$X(0) = 0$ no salt initially

$$C = -4500\sqrt{3}$$

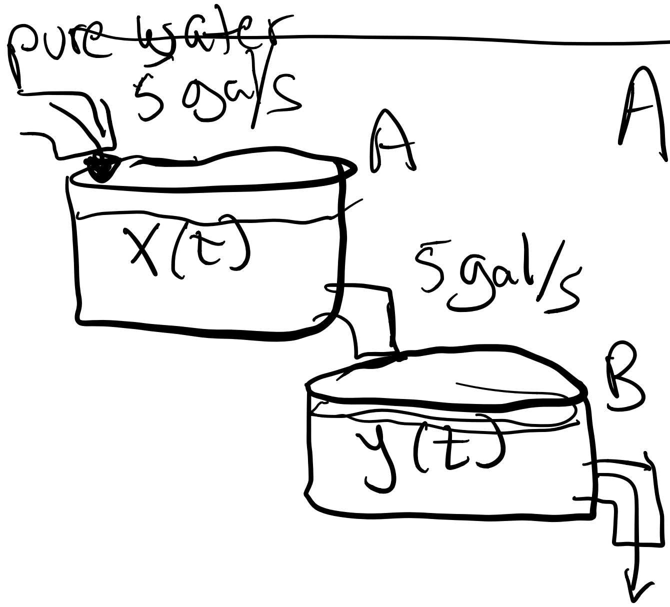
$$X(t) = 450 + 3t - \frac{4500\sqrt{3}}{\sqrt{300+2t}}$$

$V(t) = 600$ what is X?

$$V(t) = 300 + 2t = 600$$

$t = 150$ mins.

$X(t = 150) = 582$ lbs of salt



A: 100 gals water
20 lbs salt dissolve

B 200 gal water
40 lbs salt
dissolved

Tank A: concentration A:

$$\frac{X(t)}{100 \cancel{lb}/gal}$$

rate in: 0 no salt coming in

rate out: $\begin{pmatrix}\text{volume}\\ \text{out}\end{pmatrix}$(concentration)

$5\ \text{gal/s}\left(\frac{x}{100}\ \text{lb/gal}\right)$

$$\frac{dx}{dt} = 0 - \frac{x}{20}\ \text{lbs/s}$$

B rate in: $\frac{x}{20}$ lbs/s

rate out: (vol out)(concentration)

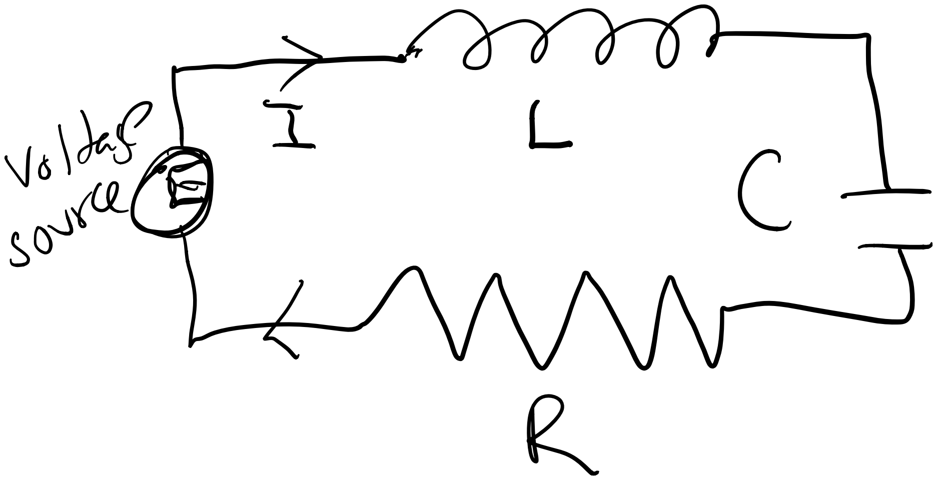
$: 5\ \text{gal/s}\left(\frac{y}{200}\right)\text{lb/gal}$

$= \frac{y}{40}\ \text{lb/s}$

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{x}{20} - \dfrac{y}{40} \\ \end{cases}$$

$$\frac{dx}{dt} = -\frac{x}{20}$$ y, x, t

Electrical Circuits



L [H]
C [F]
R [Ω]
E [V]
I [A]

1. Ohm's Law resistor impedes the flow of eletric charge. Voltage drops across a resistor, proportional to current I

$$E_R = RI$$

2. <u>Faraday's Law</u>: Current flowing through a coil produces a magnetic field which opposes any change in current
Voltage drop across coil (inductor) proprotional to the <u>rateof change</u> of the current

$$E_L = L \frac{dI}{dt}$$

3. <u>Capacitance Law</u>
Capacitor stores electric charge on two parallel plates separated by an insulator. Little to no current passes through Charge buildsup, reverses current direction

$$E_c = \frac{1}{C} Q$$

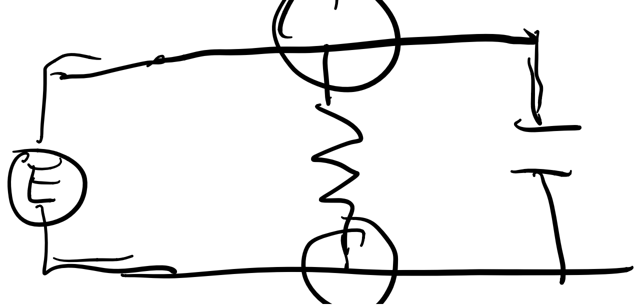
How do we apply those?

4. Kirchhoff's Voltage law

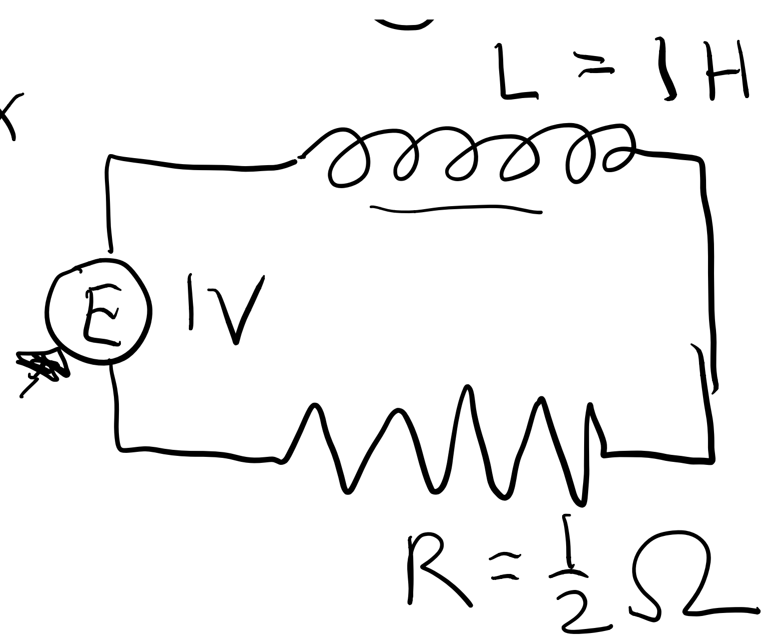
the sum of voltage drops around a closed loop in a circuit must be zero

5. Kirchhoff's current Law

Sum of currents flowing into any junction is zero



Ex



use KVL

$$E_L + E_R - E = 0$$

$$L\frac{dI}{dt} + RI = E$$

using linear methods

L=1H

$$\frac{dI}{dt} + \frac{1}{2}I = 1$$

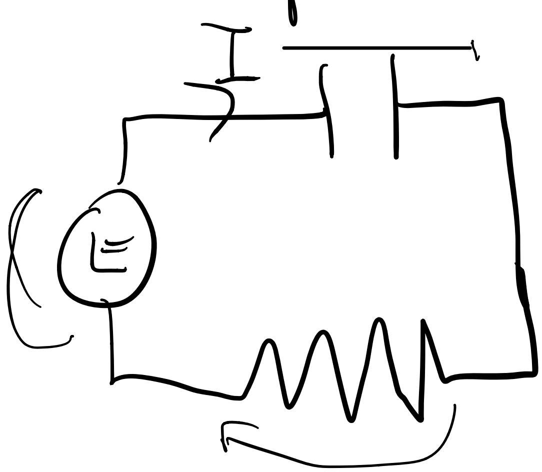
$$\frac{dI}{dt} = 1 - \frac{1}{2}I$$

$$\int \frac{dI}{1-\frac{1}{2}I} = \int dt$$

$I(0) = 0$

$$I(t) = 2\left(1 - e^{-\frac{1}{2}t}\right)$$

Example 2



$C = \frac{1}{5}F$

$R = 2\Omega$

Voltage source

$E = \cos t$ V

Find current $I(t)$

initial current

$I(0) = 0$ Amp

$$E_C + E_R - E = 0$$

$$\frac{1}{C} Q + R I - \text{cost} = 0$$

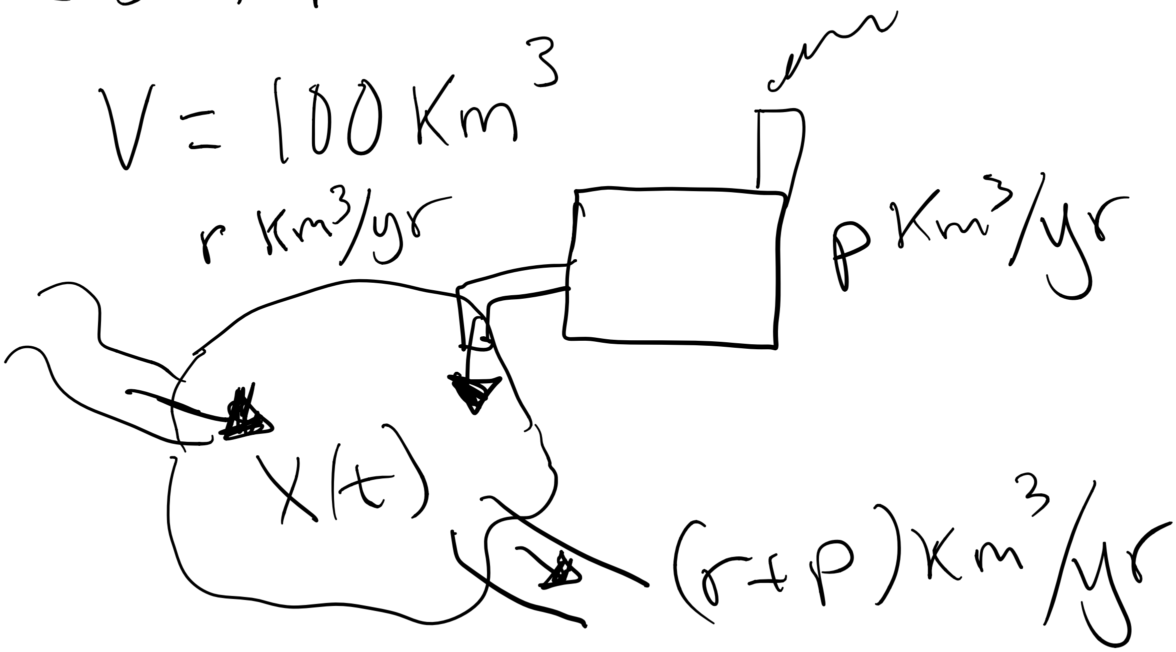
$$\frac{\Delta Q}{\Delta t} \rightarrow \boxed{\frac{dQ}{dt} = I}$$

$$\frac{1}{C} Q + R \frac{dQ}{dt} = \text{cost}$$

First order inhomogeneous equation

2.5 #9

$V = 100\ Km^3$



$$C = \frac{X(t)}{V(t)} = \frac{X(t)}{V}$$

$$\left(\frac{X}{V}\right)' + \frac{p+r}{V}\left(\frac{X}{V}\right) = \frac{p}{V}$$

$$\frac{X'}{\cancel{V}} + \frac{(p+r)X}{V^2} = \frac{p}{\cancel{V}}$$

$= C$

$$\boxed{\frac{dx}{dt} = p - \underbrace{(p+r)}\cdot\frac{x}{V}}$$

$= \text{rate in} - \text{rate out}$

$$x = C \cdot V$$

$$\frac{d}{dt}(C \cdot V) = p - (p+r)\frac{CV}{V}$$

$$\frac{\cancel{V} C'}{\cancel{V}} = \frac{p}{V} - \frac{(p+r)C}{V}$$

$$C' = p - \frac{p+r}{V} C$$