8/31/2020

Differential Equations

HW #2 due Friday @ midnight EST

Office hours Tuesday 1:30pm - 2:30pm EST

same zoom link

Last day for Add/Drop Fri Sept 4^{th} !!

Please do poll

Review:

error for 4^{th} order Runge Kutta

$$h^4 \quad 0 < h < 1$$

Solutions to Separable Equations

$$\frac{dy}{dt} = y \cdot t$$

1. Separate Variable to di-fferent Sides

$$\int \frac{dy}{y} = \int t\,dt$$

2. Integrate

$$\ln|y| + C_1 = \frac{t^2}{2} + C_2$$

$$\ln|y| = \frac{t^2}{2} + (C_2 - C_1)$$

$$\ln|y| = \frac{t^2}{2} + C$$

$$y(t) = e^{t^2/2 + C}$$

$$y(t) = e^{t^2/2} e^{C} = Ae^{t^2/2}$$

Decay rate of an unstable nucleus

$$N' = -\lambda N \qquad \lambda > 0$$

constant

$$\frac{dN}{dt} = -\lambda N$$

$$\int \frac{dN}{N} = -\int \lambda\, dt$$

$$\ln|N| = -\lambda t + C$$

$$N(t) = Ae^{-\lambda t}$$

half-life of radio active material

After 10hr, 615mg of 1000mg sample is left. What is the half life of the material?

$$\boxed{t=0 \quad N=1000\,mg}$$

$$N = Ae^{-\lambda t}$$

$$1$$

$$1000 = A\cancel{e^{-\lambda(0)}}$$

$$1000 = A$$

$$N(t) = 1000\,e^{-\lambda t}$$

$$t = 10\,hr \quad N = 615\,mg$$

$$615 = 1000\,e^{-\lambda 10}$$

$$\frac{615}{1000} = e^{-10\lambda}$$

$$\ln\left(\frac{615}{1000}\right) = -10\lambda$$

$$-\frac{1}{10}\ln\left(\frac{615}{1000}\right) = \lambda$$

$$0.04861 = \lambda$$

$$\boxed{N(t) = 1000\,e^{-0.04861\lambda}}$$

$t = ?$ $\quad N = 500\,mg$

$$N = 1000e^{-0.04861t}$$

$$500 = 1000e^{-0.04861t}$$

$$\frac{1}{2} = e^{-0.04861t}$$

$$\ln(1/2) = -0.04861 \cdot t$$

$$\boxed{14.26\text{ hours} = t}$$

Ex

$$y' = ty^2$$

$$\frac{dy}{dt} = ty^2$$

$$\int \frac{dy}{y^2} = \int t\,dt$$

$$-1 \cdot \frac{1}{y} = \frac{t^2}{2} + C$$

$$-\frac{1}{y} = \frac{1}{2}t^2 + C$$ cross multiply

$$-1 = y\left(\frac{1}{2}t^2 + C\right)$$

$$\left(\frac{2}{2}\right)\frac{-1}{\frac{1}{2}t^2 + C} = y$$

$$\frac{-2}{t^2 + 2C} = y(t)$$

$$2 \cdot C = B$$

What are separable equations?

$$\frac{dy}{dt} = \frac{g(t)}{h(y)} \qquad \frac{dy}{dt} = g(t) \cdot f(y)$$

ex: $\sin(t) \cdot \cos y$

$$e^{-t} \cdot \frac{1}{y+2}$$

not separable: $y \pm t$

The zero problem

$$\frac{dy}{dt} = g(t) f(y)$$

if $f(y) = 0$ &

$y(t) = y_0$ a constant

$$\frac{d(y_0)}{dt} = 0$$

$y' = ty^2 \qquad y(0) = 0$

$$y(t) = \frac{-2}{t^2 + 2C}$$ also a solution

$$0 = \frac{-2}{2C}$$

$$0 = \frac{1}{C}$$ no finite C solves this

So general solution doesn't always give every single solution.

$C = \infty$

Using Definite Integration

Ex Newton's law of cooling

Can of soda $40°F$

in room temp $70°F$

After 10 minutes $50°F$

What is $T(t)$?

$$\frac{dT}{dt} = -K(T-A)$$

A = room temp

$$\frac{dT}{(T-A)} = -K\,dt$$

$$\frac{dT}{T-A} = -k\,dt$$

$$T \to s$$

$$T(t=0) = T_0 \qquad t \to u$$

$$\int_{T_0}^{T} \frac{ds}{s-A} = -k\int_{t=0}^{t} du$$

$$\ln|s-A|\Big|_{T_0}^{T} = -kt$$

$$\ln|T-A| - \ln|T_0-A| = -kt$$

$$\ln\left|\frac{T-A}{T_0-A}\right| = -kt$$

$$\frac{T-A}{T_0-A} = e^{-kt}$$

$$T-A = (T_0-A)e^{-kt}$$

$$T(t) = A + (T_0 - A)e^{-kt}$$

$A = 70°F \quad T(10) = 50°F$

$$\boxed{k = \frac{1}{10}\ln 3/2 = 0.0405}$$

written steps on pg 12

Implicit Solutions

$$y' = \frac{e^x}{(1+y)} \qquad y(0) = 1$$
$$y(0) = -4$$

$$\frac{dy}{dx} = \frac{e^x}{1+y}$$

$$\int (1+y)\,dy = \int e^x\,dx$$

$$y + \frac{y^2}{2} = e^x + C$$

$$y^2 + 2y - 2(e^x + C) = 0$$

$$y^2 + 2y - 2(e^x + c) = 0$$

$$y = \frac{-2 \pm \sqrt{2^2 - 4(-2)(e^x + c)}}{2}$$

$$= \frac{-2 \pm \sqrt{4 + 4(2)(e^x + c)}}{2}$$

$$= \frac{-\cancel{2} \pm \cancel{2}\sqrt{1 + 2(e^x + c)}}{\cancel{2}}$$

$$y(t) = -1 \pm \sqrt{1 + 2(e^x + c)}$$

✓ $y(0) = 1$ $\qquad$ $y(0) = -4$

$$1 = -1 + \sqrt{1 + 2(e^x + c)}$$

$$\boxed{c = 1/2}$$

Cont on Wed

$$T - A = (T_0 - A)e^{-kt}$$

$$T(t) = A + (T_0 - A)e^{-kt}$$

$$T(t=10) = 50 \quad A = 70$$

$$T(10) = 50 = 70 + (40 - 70)e^{-k10}$$

$$50 = 70 + (-30)e^{-10k}$$

$$-20 = -30e^{-10k}$$

$$\frac{2}{3} = e^{-10k}$$

$$\ln 2/3 = -10 \cdot k$$

$$-\frac{1}{10}\ln 2/3 = k$$

$$A \ln B = \ln(B)^A$$

$$T_0 \left(-1 \ln(2/3) = \ln(2/3)^{-1} \right)$$